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Classes of Finite Solutions to the Inverse Problem of the Logarithmic Potential

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Abstract—We obtain a new class of solutions to the inverse problems of the logarithmic potential in the form of a logarithmic function of a ratio of polynomials of the same degree. We give examples of finite solvability of the inverse problems.

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INTRODUCTION

The papers on the inverse problems of the logarithmic potential have different source data which use formulas of the mathematical physics. The difference is in constant factors, which do not affect the essence of the subject. In this regard, we compare the source data in the papers of I.M. Rapoport [1, 2] and V.K. Ivanov [3, 4] in the Section 1 of this article. We establish a new form of the integral equations, whose solutions the inverse problems are reduced to. In the Section 2, we give a solution to an example connected with Ivanov's theorem [4] and give a slightly modified proof of this theorem. In the Section 3, we write out and justify a new class of solutions to the inverse problems in the form of a logarithmic function applied to a ratio of polynomials of the same degree. We give a solution to a concrete problem from this class. In the Section 4, we give examples of polynomial solutions, which complement Rapoport's results, and prove the theorem that generalizes these examples.

For geometric characteristics of the solutions found, we apply the criteria for convexity and starlikeness ([5], Ch. IV, § 5), in order to use Novikov's uniqueness theorem [6].

1. We recall that the logarithmic potential has the form

$$\int\int\limits_{D} 2\mu \ln\left[1/R(z,z_1)\right] d\sigma(z_1),$$

where $R(z,z_1)=|z-z_1|$, $d\sigma(z_1)$ is an element of the area containing point z_1, arises for cylindrical bodies filled with homogeneous gravitational mass with a density $\mu\equiv\text{const}$. Let us preliminarily characterize a vector field in the complex plane z, that is perpendicular to the cylinder axis. Denote a variable point on the cylinder axis by t, $-\infty<t<\infty$. We have

$$|\overrightarrow{F}(z)| = \int\int\limits_{D}\int\limits_{-\infty}^{\infty} \frac{\mu d\sigma(z_1)dt}{\left(\sqrt{t^2+|z-z_1|^2}\right)^2}\frac{|z-z_1|}{\sqrt{t^2+|z-z_1|^2}}, \tag{1}$$

which is the force a point z with a mass of 1 is attracted with to a cylindrical column (infinite in both directions) with a base D. Formula (1) is obtained by projection of the gravity force that is induced by

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